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***Spherically symmetric dark energy
structures in the context of Chaplygin
gas model***

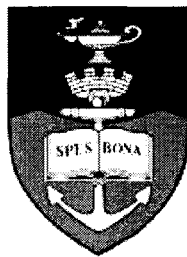
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Declaration

I, the undersigned, hereby declare that the work contained in this thesis is my own original work and has not previously in its entirety or part been submitted at any university for a degree

27 March 2006

Signature

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Abstract

This paper investigates the existence of spherically symmetrical dark energy structures under the context of Chaplygin gas. A scaling solution of the form $r^{-2/3}$ is found for the density and then we calculated the corresponding rotational curve and it turns out to be unrealistic implying that such objects don't exist. Finally we modified the equation of state of the Chaplygin gas to an equation of state of the form $P = \sigma^2 \rho - \frac{A}{\rho}$ and compared with observational data to see to what physical extent this equation works and it is determined that it works as far as a couple of hundred Mpc while the physical length of galaxies is in Kpc. implying the modified Chaplygin equation fits the observed rotational curves.

Introduction

The universe we are living is not only made of the ordinary matter we observe. Rather it is also made of two other entities called dark matter and dark energy. A very convincing evidence for the existence of dark matter comes from the study of the flat rotational curves of galaxies. The flat rotational curve of stars in the galaxy shows that there is a hidden matter which differs from the ordinary mass in its nature and this hidden mass is dark matter [1]. The existence of the second entity (dark energy) is revealed while trying to account for accelerated expansion of the universe. This entity is the major source of the force driving the current acceleration of the expansion of the universe [1].

The contribution of ordinary matter (the matter we see) to the total mass-energy budget of the universe is very small as compared to the other unseen components contributing to it. Most of the mass-energy content is contributed by dark energy and dark matter. The detailed study of the Cosmic Microwave Background (CMB) fluctuations by WMAP revealed that the breakdown of the mass-energy budget of the universe is[2-3]: 4% ordinary matter, 23% dark matter and 73% dark energy. i.e, 96% of the total content of the universe is non-baryonic matter.

Though no one knows what dark energy actually is, different models with their own strong and weak sides have been proposed. Amongst these models: the cosmological constant (Λ) or vacuum energy[1,3], scalar fields or quintessence[1,4], the brane-world cosmology model of dark energy[1,5,6] and the Chaplygin gas model[7] are the most popular ones.

The last of these models, Chaplygin gas model, deals with an exotic gas based on an equation of state: $P = -\frac{A}{\rho}$. This model interpolates between the dust and the de Sitter

eras smoothly through an intermediate regime of cosmological constant and stiff matter, which is characterized by the equation of state: $P = \rho$. The Chaplygin gas model has got a bound and positive speed of sound squared which removes the instability problem at small scale [7]. The Chaplygin gas model is a model that allows for the possibility of the unification of dark matter and dark energy. This unification happens in the context of the inhomogeneous Chaplygin gas. It unifies dark matter and dark energy in a geometric setting reminiscent of M-theory and hence according to this inhomogeneous Chaplygin gas model dark matter and dark energy are two different manifestations of the same entity [8].

The Chaplygin gas model has a strong connection with the brane world cosmology. For example it is possible to get the equation of state of a Chaplygin gas from the stabilization of branes in a black hole. It is found that matter with equation of state like that of the Chaplygin gas should be added to stabilize the brane [9].

In this paper we study the possibilities of the formation of spherically symmetrical dark energy structures in the context of the Chaplygin gas and we discuss the idea of modifying the Chaplygin gas equation of state in such a way that the rotational curve predicted by the Chaplygin gas agrees with the observations. The validity of this modified equation of state is also checked against some observational data from some selected LSB and HSB galaxy data [10].

The paper is organized as follows. In chapter one general overview of dark matter, dark energy including its different models and its responsibility for the accelerated expansion of the universe is given. In chapter two we drive the scaling solution for the density and an expression for the radius in terms of the scale factor. In chapter three we examine the implication of the solutions and expressions we got in chapter two and finally in chapter four we made conclusion.

Chapter 1

Theoretical background

1.1 Dark Matter

Observational evidences show that the universe is not only made of ordinary matter. Rather it is made also of non-baryonic matter called dark matter. This non-baryonic matter is even the dominant form of matter in the universe. The ordinary matter contributes little to the total energy-mass budget. The recent best estimate for energy density (Ω_b) of ordinary matter is $\Omega_b = 0.04 \pm 0.02$ compared to the total energy density ($\Omega=1$)[11]. Unlike ordinary matter, dark matter neither emits nor absorbs electromagnetic energy. It can only be detected through its gravitational effects on visible mass.

Accepting that gravity is the only force responsible for galactic dynamics, the most convincing evidence for the existence of dark matter comes from the discrepancy between galactic rotational curve mass prediction and their visible mass [12]. Kepler's second law tells us that the stellar rotational velocity inside a galaxy of spherically symmetric density distribution at radius r from the galactic center is given as

$$V_c(r) = \sqrt{\frac{GM(r)}{r}} \quad (1.1)$$

where $M(r)$ is the enclosed mass within the radius r .

It is clearly seen from this mathematical expression that the rotational velocity declines as $r^{-1/2}$. However, detailed studies of the 21cm emission line of neutral hydrogen (HI) from different galaxies confirms that the rotational curves of galaxies are flat. This means the rotational speed doesn't decline as the radius increases from the galactic center; rather it stays constant. This contradicts the above mentioned law governing galactic dynamics.

A convincing reconciliation between these two contradicting ideas is the mass must be linearly dependent on the radius ($M(r) \propto r$) like the case in the isothermal sphere so that the curve is flat. This in turn implies that there must be additional unseen non-luminous mass in the galaxy which increases with radius. The rotational curve mass prediction is almost ten times greater than the visible mass. This missing mass exists in form of matter whose mass to light ratio is much greater than that of the visible matter (baryonic matter) in side galaxies [1].

Another good evidence for the existence of dark matter is the cosmic microwave background (CMB) measurements and the implication of these observations for structure formation. According to the CMB observations baryonic matter alone cannot support structure formation because of their weak initial perturbation. In a baryon dominated universe, the high radiation pressure ensures that the density perturbation starts growing only after hydrogen recombines. The amplitude of this density perturbation ($\delta\rho/\rho$) found for baryons is in the order of 10^{-5} at decoupling ($z = 1100$). However, $\delta > 10^{-3}$ required to get the present day $\delta > 1$ value which is quite different from the CMB value mentioned above. The dark matter theory for structure formation solves this problem. The fact that dark matter doesn't interact with both baryonic matter and radiation, structure formation can grow even before hydrogen is recombined. Then baryons with their weak density fluctuation fit to the potential wells already created by the dark matter and give rise to the formation of all the structures seen today. Thus dark matter is necessary for explaining structure formation in the universe [13].

In addition to these two popular evidences, there are also many other ones. These include:

➤ *Velocities of galaxies in clusters*

Studying the random motion of galaxies in a cluster, it is possible to determine the dispersion velocity. These studies show that the dispersion velocity found for such system would not be possible only with observed mass rather there must be as much as ten times the luminous mass to account for the observed dispersion value. With the observed visible mass, the galaxies would have dispersed in less than one billion years and there would be no galaxy clusters in the universe [14].

➤ ***Gravitational lensing***

According to general relativity, the straight line path of light is affected by gravity and hence light paths will bend in a gravitational field. In the presence of high mass concentration the light coming from a background source is bent by this high mass concentration and the back ground light sources appear either distorted or multiply imaged. Theoretically, if the distance between the foreground and the back ground is known, it is possible to determine the mass in the region and this visible mass is short by many factors and hence the missed mass must be accounted by dark matter [14].

➤ ***Hot gas in galaxies and clusters***

Clusters of galaxies are intense sources of X-rays. Actually, the X-rays do not come from the galaxies themselves, rather from the rarefied hot gases which are at about a temperature of $10^7 K$ in the regions between galaxies. To keep this hot gas together against the thermal motion of their particle requires a huge amount of mass but the observed mass of hot gas is too small. So it requires dark matter to solve this puzzle [14].

1.2 Dark Energy

Type Ia supernovae which are considered as standard candles in observational cosmology give evidence that the expansion of the universe is currently accelerating instead of decelerating. In addition to carrying the fingerprint of the initial condition of the early universe, CMB itself gives strong support for the speeding-up of the expansion. Obviously, there should be something with a negative pressure pushing against the action of gravity, and it is even in excess to win over gravity and causing the expansion. The question is what is this something? This something driving the acceleration is called dark energy. So far there is no-clear cut idea of what dark energy is but it is believed that dark energy like dark matter has non-baryonic origin. There are different models suggested for dark matter. These are:

1.2.1 The cosmological constant(Λ) and vacuum energy

Einstein introduced cosmological constant in 1917, believing that the universe is static; later he learnt that the universe is not static and he abandoned the idea of cosmological constant. The idea of cosmological constant finally resurrected and got much attention as a model for dark energy after many years but now as a form of matter (dark energy). It is considered to be the energy of free space (vacuum) that has negative pressure and thus can induce cosmic acceleration.

The Einstein field equation, including cosmological constant is written as:

$$R_{ij} - \frac{1}{2} g_{ij} R = \frac{8\pi G}{c^4} T_{ij} + \Lambda g_{ij} \quad (1.2)$$

For a Friedmann- Robertson-Walker (FRW) universe dominated by pressureless dust (dark matter) and cosmological constant the above equation (1.2) can be rewritten in its force law form as:

$$F = -\frac{GM}{R^2} + \frac{\Lambda}{3}R \quad (1.3)$$

Unlike the first term in the right hand side of equation (1.3) the cosmological constant term represents the repulsive force playing the role of dark energy. Another more general version of equation (1.3) is

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \sum_i (\rho_i + 3p_i) = -\frac{4\pi G}{3} \sum_i \rho_i (1 + 3w_i) \quad (1.4)$$

The summation runs over all contributing forms of matter with general equation of state $w_i = p_i / \rho_i$.

Using equation (1.4) together with the equation:

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \sum_i \rho_i - \frac{k}{a^2} \quad (1.5)$$

it is easy then to see that, if the universe accelerates, the strong energy condition ($\rho + 3p \geq 0$ or $w_i \geq -\frac{1}{3}$) is violated.

The expansion of the universe makes the effect of gravity diminishes by spreading the matter in a relatively large physical space. However the expansion doesn't have the same effect in the case of vacuum energy. The vacuum PdV work done during adiabatic expansion produces the same amount energy that could fill the new volume with the same density and hence cosmological constant (vacuum energy) is a constant (a fundamental constant) and spatially uniform.

The non-zero value of the zero point energy ($E_0 = \frac{1}{4\pi} h\omega$) predicted by quantum mechanics is the physical basis for the interpretation of cosmological constant as vacuum energy density. However the vacuum energy value is divergent and this gives rise to the

well known cosmological constant problem. The diverging value of the cosmological constant in this case is:

$$\frac{\Lambda}{8\pi G} \propto \int_0^\infty \sqrt{k^2 + m^2} k^2 dk \quad (1.6)$$

where k is the wave vector.

It is possible to get a finite value by restricting the value of k only to the cutoff value of plank scale ($k_{\max} \gg \frac{2\pi m}{h}$). Even if it is possible now to get a finite value, the imposition doesn't solve the inconsistency with the very small observational value. The theoretical estimate is 120 orders of magnitude discrepant [3] which is:

$$\rho_{vac}^{obs} \sim 10^{-120} \rho_{vac}^{guess}$$

So far there is no satisfactory approach to reconcile the discrepancy of the big theoretical estimate and the small observed value but Supersymmetry gives some hope by stating that bosons and fermions contribute equally but with opposite sign, if the degree of freedom is exactly the same, so that the net vacuum energy is zero. However we don't have any proof that we are living in Supersymmetric state.

If Supersymmetry exists at all, it is broken at a certain limit: M_{susy} , and the vacuum energy is not zero in a broken Supersymmetry. In mass scale, it is almost the same order with M_{susy} .

The other approach to the cosmological problem is based on some philosophical ideas like the anthropic principle which is not completely a physical way of addressing a physical problem [1, 7].

It is known that the ratio of the vacuum energy density and matter energy densities is a function of the scale factor $a(t)$:

$$\frac{\Omega_{\Lambda}}{\Omega_M} = \frac{\rho_{\Lambda}}{\rho_M} \propto a(t)^3 \quad (1.7)$$

The implication of the above equation is at early times the matter energy density was more important than the vacuum energy density and the opposite is true at late times and there is a transition point from one to the other where the two energy densities are of the same order of magnitude.

The fact that the universe started accelerating very recently together with the above idea causes another problem: “the cosmic coincidence” puzzles. The puzzle is: how intelligent life emerges at the special time where the density of dark matter and dark energy are almost of the same order of magnitude.

Any dark energy model should have its initial condition carefully chosen so that it dominates right at the present time. Cosmological constant, as a dark energy model with unevolving equation of state, its initial condition must be properly tuned. The fine tuning issue is a very serious issue in deciding which dark energy model is accepted or not.

1.2.2 Scalar field or Quintessence

If the general relativistic description of cosmic acceleration is correct, the Friedmann equation suggests a dynamical model with a slowly decreasing dark energy density as the universe expands.

We should be able to consider another model of dark energy which is time dependent. There are many reasons why we consider evolving dark energy models. The very important ones are:

- 1 Dynamic dark energy model can evolve slowly to zero. This may allow a solution to the cosmological constant problem which causes the vacuum energy to vanish.

- 2 Dynamical models could possibly open the door to the possibility of getting dynamical solutions to the coincidence problem and even to the fine tuning problem.
- 3 And finally dynamical dark energy models could suggest radical new picture of the overall history of the universe

Scalar field (quintessence) is one of the logical and simplest possibilities of such models with a dynamic equation of state. For a homogeneous field ($\nabla\phi \approx 0$), the energy density which is contributed only from the kinetic energy and potential energy terms is given as:

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) \quad (1.8)$$

And the equation of motion in an expanding universe is:

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0 \quad (1.9)$$

If the first term of the left hand side of the above equation is negligible (or if V is quite flat), then we can have a solution for ϕ varies slowly with time and stays almost constant throughout space. The energy density for such system is:

$$\rho_\phi \approx V(\phi) \approx \text{Constant}$$

The fact that ϕ varies slowly with time such scalar fields could be considered as a good candidate for dark energy model.

This scalar field model encompasses a wide variety of possible candidates for dark energy each with equation of state $-1 \leq w \leq 0$.

A scalar field model that could address the coincidence problem pretty well is the k-essence model. The total idea of K-essence is based on the fact that matter-radiation equality is a recent phenomenon (at least on a logarithmic scale). It is a model where the action for the scalar field has not potential energy term but only kinetic energy term with

a little bit of modification from the usual conventional kinetic energy ($K = \frac{1}{2}(\dot{\phi})^2$). The kinetic energy is taken as:

$$K = f(\phi)g(\dot{\phi})^2 \quad \text{where } f \text{ and } g \text{ are functions determined by the model.}$$

For particular values of the functions f and g the k-essence tracks the evolution of the total radiation density during the radiation dominant epoch. That means the k-essence behaves as if it is radiation ($w = 1/3$) and the energy density declines in parallel (from the lower side) with the dominant radiation energy density. Such models are called tracker models. So this tells that the k-essence energy density is negligibly smaller than the radiation energy density throughout the radiation-dominated era. But as soon as the matter domination era starts, the energy density of the k-essence drops down and then a new matter dominated attractor solution follows. Now the k-essence tracks the matter-dominated energy density by staying constant like cosmological constant ($w = -1$) and overtake the matter domination energy density after. It is worth noting at this point that k-essence doesn't dominate immediately after the matter domination starts rather it dominates after a while. However after it has stayed constant, it doesn't take long in the cosmic time scale to dominate all the other two components so that it pushes the universe to its accelerating phase [1,4]. The general story is demonstrated in figure 1 below:

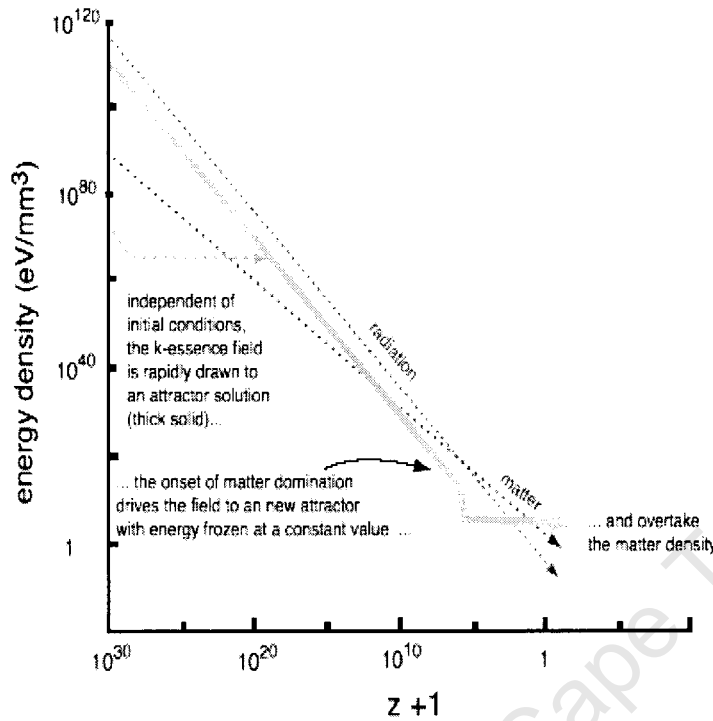


Figure 1 A graph showing the matter, the radiation and the k-essence energy density as a function of red shift (taken from a quintessential introduction to dark energy [4]).

The k-essence model addresses the coincidence problem beautifully. Why did the cosmic acceleration and the emergence of intelligent life form coincide? According to the k-essence model both phenomenon occur at the onset of matter domination and the k-essence term acts a negative pressure after matter-radiation equality. This implies that it can dominate the other two components after matter has dominated the universe for a certain time, nearly during the present time in cosmic time scale. The emergence of intelligent life on the other hand is the consequence of matter domination because the formation of planets, stars, galaxies and other large scale structures are the result of matter domination. Therefore k-essence is a good non-anthropic (non philosophical) dynamical dark energy model that could address the coincidence problem very well.

1.2.3 The brane-world cosmology and dark energy

Another completely different approach to the present acceleration of the universe is braneworld cosmology (higher dimensional analysis) approach. It is the description of the universe in which the observable 3-dimensional brane embedded in a higher-dimensional (usually four-dimensional) bulk. Braneworld cosmology allows a wider range of possibilities for dark energy. Some braneworld models suggested that dark energy and its consequence, the acceleration of the universe, are transitory phenomena. According to them, the universe will re-enter again to a matter dominated era after the currently accelerating era. [5].

Most works in this area have come after the paper by Randall and Sundrum [6]. This model (RS model) is similar in many aspects with general relativity, except at early stages of the development of the universe. The matter fields are confined in the brane whereas the carrier of the gravity propagates from the brane to the bulk and hence the gravity goes beyond the brane [1]. The RS model takes into consideration only the brane and the bulk cosmological constants to build the brane action.

There is also a generalized RS model. This model includes the effect of the scalar curvature term in the action of the brane in addition to the brane and the bulk cosmological constant terms. An important property of this braneworld model is the equation of state for the dark energy could be $w < -1$ and it also exhibits the property of being transient to another matter dominated era. This model could provide a good opportunity to reconcile string theory/M-theory with the currently accelerating universe because a transiently accelerating universe doesn't have an event horizon [5].

The braneworld models proposed by Deffayet, Dvali and Gabadadze (DDG) accounts for the acceleration of the universe in a completely different way. Unlike the above two

braneworld models this one considers only the scalar curvature in the action so that the equation for the action looks:

$$S = M^3 \int_{bulk} R + m^2 \int_{brane} R + \int_{brane} L_{matter} \quad (1.10)$$

where M is the 5-dimensional Planck mass and m is the 4-dimensional Planck mass.

The corresponding Hubble parameter is:

$$H = \sqrt{\frac{8\pi G \rho_m}{3} + \frac{1}{l_c^2} + \frac{1}{l_c}} \quad (1.11)$$

where $l_c = m^2/M^3$ is a new length scale determined by m and M .

The model surprisingly throws away dark energy from the total picture of searching a possible explanation to the late time acceleration of the expansion of the universe. According to this model because of the 4-diminsional Ricci scalar term induced on the brane an observer on the brane measures a four dimensional Newtonian gravity at distances shorter than l_c . At distances larger than l_c , however, the 5-dimensional force law dominates and gravity spreads into extra dimensions. Consequently the whole thing is not any more controlled by the 4-dimensional Newtonian gravity rather it is controlled by a 5-dimensional force law. Hence gravity is weaker at cosmic distances and this dramatic change affects the whole story of the accelerated expansion and gives a different picture [1,5,15].

A general braneworld model which incorporates both RS and DDG models is described by the following action:

$$S = M^3 \int_{bulk} (R - 2\Lambda_b) + \int_{brane} (m^2 R - 2\sigma) + \int_{brane} L_{matter} \quad (1.12)$$

For especial cases where $\Lambda_b = \sigma = 0$, the action takes the form of the action of DSS model and for $m = 0$ it takes the form of the RS model. Like many other class of braneworld models this model also predicts a late time accelerating universe [1].

1.2.4 The Chaplygin gas model

Chaplygin gas model is a simple dark energy model with a perfect fluid system having negative pressure [7]. The theory has been tested against different observational data and proved to be consistent. Some of the observational data it is tested against are: high precision cosmic microwave data [16], the supernova data [17] and gravitational lensing [18]. This model is well known for its explanation for the transition from the decelerated expansion to the present accelerated expansion and for its proposed unified phenomenological description of dark energy and dark matter.

The equation of state of the Chaplygin gas is:

$$P = -\frac{A}{\rho} \quad (1.13)$$

where A is a positive constant, ρ is the energy density and P is the pressure.

One good thing about this model is that it gives a large group of symmetry to the corresponding Euler equations and hence makes the Euler equations integrable in (1+1) spacetime dimensions.

It can easily be seen from the equation of state that the Chaplygin model has got a bound and positive speed of sound i.e.,

$$c_s^2 = \frac{\partial P}{\partial \rho} = \frac{A}{\rho^2} \quad (1.14)$$

This positive value of the sound speed squared alleviates the instability problem at small scale.

Another possibility in the context of Chaplygin gas is a generalized Chaplygin gas model [19] which is given as:

$$P = -\frac{A}{\rho^\alpha}, \quad 0 < \alpha \leq 1 \quad (1.15)$$

and a further possibility is a two-fluid model. This is a model combining the Chaplygin gas with dust-like matter [20], and it addresses the coincidence problem with a plausible explanation [20].

1.2.4.1 FRW cosmology with the Chaplygin gas

Consider a homogenous and an isotropic universe with a metric given as

$$ds^2 = dt^2 - a^2(t)dl^2 \text{ where } a \text{ is the scale factor and } dl^2 \text{ is the length element.}$$

And then expansion rate (the Hubble constant), the energy density (ρ) and the spatial curvature κ are related by the Friedmann equation as

$$H^2 = \rho - \frac{\kappa}{a^2} \quad (1.16)$$

where $H = \frac{\dot{a}}{a}$ and $\frac{8\pi G}{3} = 1$.

This corresponding energy conservation equation is

$$d(\rho a^3) + P d(a^3) = 0 \quad (1.17)$$

This conservation equation, combined with the equation of state for the Chaplygin gas, leads to the expression

$$\rho = \sqrt{A + \frac{B}{a^6}} \quad (1.18)$$

where B is an integration constant.

This very important equation describes the transition from a matter dominated universe to a de Sitter universe. For a very small value of scale factor a , $A \ll \frac{B}{a^6}$ and hence the density is considerably high and can be approximated as

$$\rho \sim \frac{\sqrt{B}}{a^3} \quad (1.19)$$

This means that, for a small value of the scale factor which actually corresponds with the early times, the Chaplygin gas behaves like matter.

However, for large value of the scale factor a which corresponds with late times, the second term in the square root of the density equation can be neglected and as a result the density is independent of time and is given as

$$\rho \sim \sqrt{A} \quad (1.20)$$

The corresponding pressure is also independent of time and equal to $-\sqrt{A}$. This leads to an empty universe with a cosmological constant type scenario (i.e. a de Sitter universe) at late times. Thus this model interpolates between these two eras smoothly through an intermediate regime of a cosmological constant and stiff matter, which is characterized by the following equation of state: $P = \rho$.

Therefore, the Chaplygin gas behaves like a pressureless matter at early times and like a cosmological constant later which is consistent with the generally accepted picture of the universe [7].

The generalized Chaplygin gas model version of the relation between the scale factor and the density is

$$\rho = \left(A + \frac{B}{a^{3(1+\alpha)}} \right)^{\frac{1}{1+\alpha}} \quad (1.21)$$

The normal Chaplygin equation is a special case ($\alpha = 1$) of the generalized equation. The generalized Chaplygin gas model, like the normal Chaplygin gas model ($\alpha = 1$), evolves first as dust and then as cosmological constant at late times. However, in its intermediate stage it behaves as a mixture of cosmological constant and a perfect fluid of equation of state: $P = \alpha\rho$. The parameter α in the generalized Chaplygin equation makes this model more flexible for comparison against observations.

1.2.4.2 The brane connection

The Chaplygin gas model has a strong connection with the brane world cosmology. For example, it is possible to get the equation of state of a Chaplygin gas from the stabilization of branes in a black hole. It is found that matter with equation of state like that of the Chaplygin gas must be added to stabilize the brane [9].

To see this connection let us consider the embedding of a (3+1)-dimensional brane in a (4+1)-dimensional bulk and then starting from a Lagrangian density of a massive complex scalar field Φ which could be given as

$$L = g^{\mu\nu} \Phi_{,\mu}^* \Phi_{,\nu} - V(|\Phi|^2), \quad (1.22)$$

with the scalar field $\Phi = (\phi / \sqrt{2}m) \exp(-im\theta)$.

The brane is described by the coordinate $Y^M = (x^\mu, Y^4)$ and the induced metric on the brane is

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} - \theta_{,\mu} \theta_{,\nu} \quad (1.23)$$

where $\theta(x^\mu)$ is a scalar field describing the embedding of the brane on the bulk.

The corresponding action of the brane can be put as

$$S_{brane} = \int d^4x \sqrt{-\tilde{g}} (-f^4 + \dots) = \int d^4x \sqrt{-g} \sqrt{1 - g^{\mu\nu} \theta_{,\mu} \theta_{,\nu}} (-f^4 + \dots) \quad (1.24)$$

where f^4 stands for the brane tension and the dots for other possible contributions.

Considering the fact that the variation of ϕ corresponds to scale greater than m^{-1} , we can apply Thomas-Fermi approximation $\phi_{,\mu} \ll m\phi$. Using this approximation, the resulting Lagrangian density can be put as

$$L = \frac{\phi^2}{2} g^{\mu\nu} \theta_{,\mu} \theta_{,\nu} - V(\phi^2/2). \quad (1.25)$$

The equations of motions derived from this Lagrangian density are

$$g^{\mu\nu} \theta_{,\mu} \theta_{,\nu} = V'(\phi^2/2) \quad \text{and} \quad (\phi^2 \sqrt{-g} g^{\mu\nu} \theta_{,\nu})_{,\mu} = 0 \quad (1.26)$$

For $V' > 0$, the field θ is considered as velocity field. That means $U^\mu = \frac{g^{\mu\nu} \theta_{,\nu}}{\sqrt{V'}}$ so that

$U^\mu U_\mu = 1$ on the mass shell.

The energy momentum tensor has the form of a perfect fluid with its density and pressure respectively given as

$$\rho = \frac{\phi^2}{2} V' + V \quad (1.27)$$

$$\text{and} \quad P = \frac{\phi^2}{2} V' - V \quad (1.28)$$

Combining the (1.27) and (1.28), it is possible to get

$$d \ln \phi^2 = \frac{d(\rho - P)}{\rho + P} \quad (1.29)$$

This equation (1.29) together with the generalized Chaplygin gas equation of state(1.13), gives rise to

$$\phi^2(\rho) = \rho^\alpha (\rho^{1+\alpha} - A)^{(1-\alpha)/(1+\alpha)}. \quad (1.30)$$

Finally, combining (1.27) to (1.30) with the Lagrangian density, it is now possible to show the brane connection of the Chaplygin gas. The resulting Lagrangian density is then the same as the generalized Born-Infeld Lagrangian density which can be written as

$$L_{GBI} = -A^{1/(1+\alpha)} \left[1 - \left(g^{\mu\nu} \theta_{,\mu} \theta_{,\nu} \right)^{(1+\alpha)/2\alpha} \right]^{\alpha/(1+\alpha)} \quad (1.31).$$

For $\alpha = 1$, it is possible to reproduce the Born-Infeld Lagrangian density which reads as

$$L = -\sqrt{A} \sqrt{1 - g^{\mu\nu} \theta_{,\mu} \theta_{,\nu}} \quad (1.32)$$

This Lagrangian describes the embedding of a (3+1) space time in a (4+1) bulk [9,21].

At this stage it is important to mention the inhomogeneous Chaplygin gas. This inhomogeneous Chaplygin gas model unifies dark energy and dark matter in a geometric setting reminiscent of M-theory.

The work on inhomogeneous Chaplygin gas model by B.Neven, G.Tupper and D.Viollier [8] shows that the parameter $b(a)$, the function describing evolution of the density contrast, stops growing with a and stays constant starting from $a = 1$ and they concluded that at the location where structure is formed the Chaplygin gas behaves like dark matter and in the voids it behaves like dark energy. Therefore according to this model dark matter and dark energy are two different manifestation of the same entity.

Chapter 2

Structure equations and solutions

2.1 Derivation of the scaling solution

Consider a spherically symmetric (invariant about the center of symmetry) dark energy in the context of the Chaplygin gas model, with a space-time metrics given bellow by

$$ds^2 = e^\alpha dt^2 - e^\beta dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad (2.1)$$

where α and β are functions of the radius r .

This line element then gives the following two field equations which are expressions for the density and the isotropic pressure

$$e^{-\beta} \left(\frac{\beta'}{r} - \frac{1}{r^2} \right) + \frac{1}{r^2} = 8\pi G T_0^0 = 8\pi G \rho \quad (2.2)$$

$$\text{and} \quad e^{-\beta} \left(\frac{\alpha'}{r} + \frac{1}{r^2} \right) - \frac{1}{r^2} = 8\pi G T_1^1 = 8\pi G P \quad (2.3)$$

Combining these two equations, we have:

$$\alpha' + \beta' = 8\pi G r e^\beta (P + \rho) \quad (2.4)$$

Here the primes represent derivatives with respect to r .

We now choose

$$e^\beta = \frac{1}{1 - \frac{2GM}{r}} \quad (2.5)$$

The derivative of the enclosed mass with respect to the radius r is

$$M' = 4\pi r^2 \rho \quad (2.6)$$

Solving for α' from equation (4) and with some modifications we arrive at

$$\alpha' = 2Ge^\beta \left(\frac{M}{r^2} + 4\pi P \right) \quad (2.7)$$

while the conservation equation $T_{\mu;\nu}^\nu = 0$ yields

$$P' + \frac{\alpha'}{2}(\rho + P) = 0 \quad (2.8)$$

Substituting the expression for e^β and α' from equations (2.5) and (2.7) in to equation (2.8) and rearranging the terms, we arrive at

$$\frac{P'}{P + \rho} + \frac{G \left(\frac{M}{r^2} + 4\pi P \right)}{1 - \frac{2GM}{r}} = 0 \quad (2.9)$$

Equations (2.5) and (2.9) are generalizations of the equilibrium Euler-Poisson equations.

Indeed, for small values of $\frac{P}{\rho}$, $\frac{r^3 P}{M}$ and $\frac{2GM}{r}$, it is possible to have good approximation of equation (2.9) as

$$r^2 \frac{P'}{\rho} + GM = 0 \quad (2.10)$$

Finally, combining equation (2.10) with equation (2.6), we can get

$$\frac{1}{r^2} \left(r^2 \frac{P'}{\rho} \right)' + 4\pi G \rho = 0 \quad (2.11)$$

This is a useful approximate equation to find a scaling solution for any arbitrary equation of state.

For our particular case we want to use this equation to get a scaling solution (analogous to the isothermal sphere) for a Chaplygin gas with equation of state $P(\rho) = -\frac{A}{\rho}$.

Let us assume at this point that the density profile we are looking is of the form :

$$\rho(r) = \frac{C}{(H_o r)^\gamma} \quad (2.12)$$

where H_o is the present day Hubble constant which corresponds to the critical density

(ρ_c) as $H_o^2 = \frac{8\pi G}{3} \rho_c$, C and γ being constants to be determined.

Plugging the ansatz (equation (2.12)) into equation (2.11) yields

$$-\gamma(1+2\gamma)\frac{A}{C^2}H_o^{2\gamma}r^{2\gamma-2} + 4\pi GCH_o^{-\gamma}r^{-\gamma} = 0$$

Matching the powers $2\gamma - 2 = -\gamma$ gives $\gamma = 2/3$ and

matching the coefficients $-\gamma(1+2\gamma)\frac{A}{C^2}H_o^{2\gamma} = 4\pi GCH_o^{-\gamma}$ yields

$$C = \left[\frac{28}{27} A \rho_c \right]^{\frac{1}{3}} \quad (2.13)$$

Having determined γ and C , we need to fix the constant A . To fix A , let us consider a Chaplygin gas dominated with FRW model. According to this model the square of Hubble parameter is given as

$$H^2 = \left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho_c = \frac{8\pi G}{3} \sqrt{A + \frac{B}{a^6}}$$

The present day critical density $\rho_c = \sqrt{A+B}$ for the fact that $a = 1$ at present.

The critical density can also be expressed in terms of the matter density (Ω) at high

redshift ($a \ll 1$) as $\rho_c = \frac{\sqrt{B}}{\Omega}$.

The constant A can then be found to be

$$A = \rho_c^2 (1 - \Omega^2)$$

Therefore,

$$\rho(r) = \frac{\rho_c \bar{C}}{(H_o r)^{2/3}} \quad (2.14)$$

$$\text{where } \bar{C} = \left[\frac{28}{27} (1 - \Omega^2) \right]^{\frac{1}{3}}$$

To give some insight of the numerical value of \bar{C} , it is around 0.99 at $\Omega = .27$.

Because of the assumption taken to approximate equation (2.9), the validation of this solution is subject to

$$\left| \frac{P(r)}{\rho(r)} \right| = \frac{A}{[\rho(r)]^2} = (1 - \Omega^2)^{\frac{1}{3}} \left[\frac{27}{28} H_o r \right]^{\frac{4}{3}} \ll 1 \text{ or essentially } r \ll H_o^{-1}.$$

It is possible to arrive at the same scaling solution using a Newtonian hydrodynamics approach. This approach is based on the hydrodynamic equations

$$\frac{\bar{\nabla} P}{\rho} + \bar{\nabla} \phi = 0 \quad \text{and} \quad \nabla^2 \phi = 4\pi G \rho,$$

together with the generalized Chaplygin gas equation of state. Here we assume the same form of scaling solution as in the relativistic case. Finally, matching the powers and the coefficients in both expressions as in the relativistic case is made to get the scaling solution. As there is no approximation involved in the calculation, the resulting scaling solution is exact. Thus the relativistic case is more general and it can be condensed to the Newtonian one for some special cases.

It can be seen from the density expression in equation (2.14) that the spherically symmetrical dark energy is extremely dense. For example, on a scale of 100Kpc it would have a density of order of $10^3 \rho_c$.

2.2 The matching condition

The metric inside the spherical symmetrically Chaplygin gas is the Schwarzschild metric, and the back ground fluid is a FRW (Friedmann Robertson and Walker) metric fluid. The spherically symmetric dark energy is embedded in this background fluid. For this spherically symmetric dark energy to fit properly in to the background, its Schwarzschild metric and the background FRW metric must match smoothly.

The matching of the two space-time across a three-surface discontinuity (at the boundary surface) must satisfy the Darmois-Israel junction condition. The Darmois junction condition involves the matching of:

➤ ***Intrinsic curvature from both sides of the boundary***

Avoiding unnecessary singularities require the introduction of the same coordinate system at both sides of the coordinate system. This means

$$g_{\alpha\beta 1} - g_{\alpha\beta 2} = 0 \quad (2.15)$$

The above equation insures the equality of the intrinsic curvature on both sides of the boundary surface.

➤ ***Extrinsic curvature from both sides of the boundar:***

The tensor $K_{\alpha\beta}$ must also be the same on both sides of the boundary surface as another requirement to the smooth matching. This is

$$K_{\alpha\beta 1} - K_{\alpha\beta 2} = 0. \quad (2.16)$$

This equation then demands the equality of the extrinsic curvature again on both sides of the boundary surface.

The continuity of the extrinsic curvature and the intrinsic curvature across the boundary surface together imply the continuity of the fluid pressure from both sides [22-23] which is

$$P_{FRW} = P_{SCW}, \quad (2.17)$$

where P_{FRW} and P_{SCW} stand for the Friedmann Robertson Walker and Schwarzschild metric pressures respectively.

The fact that the two pressures are equal implies that the densities in the two regions are also equal, because we are dealing with the same Chaplygin gas equation of state both inside and outside the spherically symmetric dark energy. Hence it is possible to write

$$\rho_{FRW} = \rho_{SCW}. \quad (2.18)$$

The density profile inside the spherical system (with Schwarzschild metric) is already determined in equation (2.6). However the outside fluid density profile is different as the metric is FRW rather than Schwarzschild. The outside density at high red shift ($a \ll 1$) is then given as

$$\rho_{FRW} = \frac{\rho_c \Omega}{a^3} \text{ for } a \ll 1.$$

Plugging in the corresponding densities into (2.18), we get

$$\frac{\rho_c \tilde{C}}{(H_o r)^{2/3}} = \frac{\rho_c \Omega}{a^3} \quad (2.19)$$

Solving for the radius r in terms of the scale factor a and the constant B , we arrive

$$r(t) = \tilde{B} a^{9/2} \quad (2.15)$$

where the constant $\tilde{B} = H_o^{-1} \left(\frac{\tilde{C}}{\Omega} \right)^{3/2}$.

Chapter 3

Discussion

The density profile we determined is proportional to $r^{-2/3}$ and from it we can determine the enclosed mass in a very simple way which is

$$m(r) = \int 4\pi r^2 \rho dr \quad (3.1)$$

Plugging the density expression into equation (3.1) and doing the integration, we find the enclosed mass to be proportional to $r^{7/3}$. Now once the enclosed mass is determined it is possible to calculate the rotational speed using equation (1.1).

After putting the mass into this equation, the resulting rotational speed inside the spherically symmetrical dark energy (it could be a halo inside a galaxy or even a certain spherical patch inside a galaxy) is proportional to $r^{2/3}$, which has not been observed so far. This relation completely contradicts the flat rotational curve observed for galaxies.

However, it is possible to fix this problem by modifying the pressure and the density relation [24]. This can be done by combining a Chaplygin gas pressure with a pressure that depends on density linearly, i.e.

$$P = \sigma^2 \rho - \frac{A}{\rho} \quad (3.2)$$

According to this modified pressure, the body would have a flat rotational curve at high density, because at such densities only the first term is dominant. The problem actually comes when the density is too low. In that case the second term (the Chaplygin gas term) is dominant and the same non-flat rotational curve problem is faced.

It is possible to determine the maximum distance to which the effect of the first term extends (the point where the effect of the first term stops and the effect of the second term starts) and see whether the real rotational curve extends as far as that critical point or not. A good approximation for the critical point is to take the critical point at the point where the pressure is neither high nor low. That is the point where the two pressures are equal. We can have then the following equation:

$$\sigma^2 \rho_1 = \frac{A}{\rho_2} \quad (3.3)$$

where ρ_1 and ρ_2 are the densities corresponding to the equation of state $P = \sigma^2 \rho$ and that of the Chaplygin gas $\rho = \frac{\rho_c \tilde{C}}{(H_0 r)^{2/3}}$ where $\tilde{C} = \left[\frac{28}{27} (1 - \Omega^2) \right]^{1/3}$, respectively.

The density and the rotational velocity corresponding to the equation of state ($P = \sigma^2 \rho$) of the first part in the right hand side of equation (3.2) are given respectively as [25]

$$\rho_1 = \frac{\sigma^2}{2\pi G r^2} \quad \text{and} \quad V_c = \sigma \sqrt{2}. \quad (3.4)$$

The constant σ can be fixed by comparing the second equation with the value of the circular velocity from a real rotational curve.

Plugging in the corresponding values of the densities and the value of the constants A and the constant σ into equation (3.3) and finally solving for r_c (the crossover radius) we get the expression

$$r_c = \left[\frac{V_c^4 \left(\frac{28}{27} (1 - \Omega^2) \right)^{1/3}}{8\pi G H_0^{2/3} \rho_c (1 - \Omega^2)} \right]^{3/8}, \quad (3.5)$$

where V_c is the rotational velocity which varies from galaxy to galaxy.

To give some numerical insight on the comparison between the crossover radius and the real radius of some selected LSB (low surface brightness) and HSB (high surface brightness) galaxies, the crossover radius is calculated using the above formula for both LSB and HSB galaxies with their observed rotational velocity. A value of 71km/s/Mpc and 0.28 are taken for the Hubble constant (H_0) and Ω respectively.

The real radius, the rotational velocity and the corresponding calculated value of the crossover radius for both LSB and HSB galaxies are given in table 1 and table 2 in the appendix as de.Block and McGaugh mentioned in their paper [26].

As it can be seen from table 1 and table 2 in the appendix, the real radius of both the LSB and HSB galaxies is Kpc. However, the corresponding crossover radius for each galaxy is in Mpc. This modified equation of state then works as far as a couple of hundreds Mpc. There is a huge difference between the real radius and the crossover radius. This means that the point at which the modified equation of state fails to hold, is out of the physical extent of these galaxies and hence the modified Chaplygin equation of state can be applicable in these galaxies and can explain the flat rotational curves very well. Therefore this equation could be a good alternative to the Chaplygin gas equation as it could possibly alleviate the rotational curve problem that Chaplygin gas causes at low density regimes for spherically symmetric dark energy structure.

It is clear that σ must be a constant that is not a characteristic of a certain galaxy. However the rotational velocity V_c is different from galaxy to galaxy and since σ depends on the rotational velocity as: $\sigma = V_c / \sqrt{2}$, σ is then different from galaxy to galaxy. This is the main problem to generalize such models as one theory which governs all sorts of galaxies.

We now discuss the basic question whether such spherically symmetrical dark energy structure forms or not in the first place.

Bertolami and Paramos in their recent paper [27] studied a spherically symmetrical dark energy structures(they call it dark energy) using a polytropic equation of state of negative index and argued that there are conditions in which such objects can form due to density fluctuation in the background generalized Chaplygin gas. The condition that helps initial fluctuation to grow according to them is that the sound velocity at the surface of the dark star must be less than the initial expansion velocity the dark star.

To see the validity of their argument, the issue is addressed with already made basic ideas of structure formation that are applicable to Chaplygin gas model.

Generally a non-vanishing speed of sound is a major problem for structure formation in all unified models of dark energy and dark matter. Such systems have a characteristic scale (the sonic horizon) below which the pressure frustrates gravity from its effect and hence structure formation is prevented. Small perturbations of scales below this characteristic scale dies off without any noticeable effect.

Combining the continuity and the Euler-Poisson equations together for the case of vanishing shear and rotation and subtracting the background and finally changing the variable from t to a , one gets an equation of the form below [28]

$$a^2 \delta'' + \frac{3}{2} a \delta' - \frac{3}{2} \delta(1 + \delta) - \frac{4}{3} \frac{(a \delta')^2}{1 + \delta} - \frac{1 + \delta}{a^2 H^2} \frac{\partial}{\partial x_i} \left(\frac{c_s^2}{1 + \delta} \frac{\partial \delta}{\partial x_i} \right) = 0, \quad (3.6)$$

where δ is the density contrast which is given as $\frac{\rho - \bar{\rho}}{\bar{\rho}}$, c_s is the sound speed which is

given as $c_s^2 = \frac{dP}{d\rho} = \frac{A}{\rho^2}$ in the context of Chaplygin gas model.

The linear solution for the perturbative density contrast of the equation (3.6) above which is discussed well by Fabris et al [29] is given as

$$\delta_{per}(k, a) \propto a^{-1/4} J_{5/14}(d_s k), \quad (3.7)$$

where $J_\nu(z)$ is the Bessel function and k is the comoving wave number and d_s is the sonic horizon. This means $\delta_{per} \sim a$ for $d_s k \ll 1$ and oscillates with a decaying amplitude when $d_s k \gg 1$. This is because the non-zero speed of sound and hence non-zero pressure opposes the effect of gravity below that characteristic (sonic horizon) scale. This characteristic scale in the context of Chaplygin gas model is normally given as [28]

$$d_s = \int_0^a \frac{c_s}{a^2 H} da = \frac{2}{7} \frac{(1 - \Omega^2)^{1/2}}{\Omega^{3/2}} \frac{a^{7/2}}{H_0} \quad (3.8)$$

where $c_s = \frac{\sqrt{A}}{\rho}$ and $\Omega = \sqrt{B/(A+B)} = \sqrt{B}/\rho_{cr}$

The sonic horizon has a value of about 0.18 Mpc distance at a redshift of about 20. This distance is much greater than the size of the biggest galaxy ever known.

It is necessary to investigate equation (3.6) beyond the linear regime to conclude if initial perturbation can grow to the extent that they are powerful enough to give rise to gravitational condensation or not. Generally in the nonlinear region there is a possibility, though very small, that initial perturbations can grow undiminished. Bilic, Lindebaum, Tupper and Viollier have concluded in their paper [28]: "In contrast to linear theory, where for any R the acoustic horizon will eventually stop δ_R from growing irrespective of the initial value of the perturbation, here, for initial $\delta_R(a_{dec})$ above a

certain threshold, $\delta_R(a) \rightarrow \infty$ at finite a just as in the dust model. Conversely, at sufficiently small $\delta_R(a_{dec})$, the acoustic horizon can stop $\delta_R(a)$ from growing even in a mildly nonlinear regime.”

However, they also showed in the same paper that the fraction of Chaplygin gas that condensates due to this infinitely high perturbation is only 1% [28]. This is a very small fraction to conclude that Chaplygin gas is in favor of structure formation.

Being a Chaplygin gas system with a non-zero speed of sound ($c_s^2 = \frac{dP}{d\rho} = \frac{A}{\rho^2} \neq 0$), the spherically symmetric dark energy we are discussing is governed by the above general perturbation principle of Chaplygin gas.

This implies that structure formation is impossible as the effect of gravity is opposed by the pressure. The fact that structure formation is impossible contradicts with the existence of spherically symmetric dark energy structures (dark stars) from the first place.

Chapter 4

Conclusion

Applying the Einstein field equation with a Swarzschild space-metric for a spherically symmetrical dark energy in the context of Chaplygin gas equation of state, a scaling solution of the form:

$$\rho(r) = \frac{\rho_c \tilde{C}}{(H_o r)^{2/3}} \text{ where } \tilde{C} = \left[\frac{28}{27} (1 - \Omega^2) \right]^{1/3}$$

is found in the Newtonian approximation where $P \ll \rho$.

It can be seen from the density profile above that the spherically symmetrical dark energy is extremely dense. For example on a scale of 100Kpc it would have a density of order $10^3 \rho_c$.

For this spherically symmetric dark energy to fit to the back ground FRW metric fluid, the metric inside (Schwarzschild) and metric outside (FRW) must match smoothly. The matching requires the densities on both sides to be equal and this equality results in the

equation $r(t) = \tilde{B} a^{9/2}$ where $\tilde{B} = H_o^{-1} \left(\frac{\tilde{C}}{\Omega} \right)^{3/2}$.

The rotational curve for this spherically symmetrical dark energy is calculated to be proportional to $r^{2/3}$ which is unrealistic in comparison with the flat rotational curves of galaxies known so far.

Actually this problem can be avoided by modifying the Chaplygin equation of state to

$$P = \sigma^2 \rho - \frac{A}{\rho}.$$

At high density the first term dominates and the resulting rotational curve

is consistent with the flat rotational curves observed for galaxies but at low density the rotational curve is unrealistic. Two types of galaxies (LSB and HSB) are considered to compare the real radius of these sample galaxies with the radius to which the modified equation of state is working. Fortunately, the point from which this unrealistic rotational curve starts to dominate is completely out of the physical size of the sample galaxies considered and hence this modified equation of state looks successful to explain the spherically symmetrical dark energy rotational curve. The problem is as the constant σ is different from galaxy to galaxy; it doesn't have one general equation which holds for all galaxies.

Do such objects exist anyways? The structure formation study of Chaplygin gas shows that the probability of getting an initial density fluctuations which can cause large scale structures to form is only 1% and hence according to that study such dark energy structures don't form.

Appendix

Name	Vmax	Rmax(Kpc)	Rcross(Mpc)
F561-1	52	10.1	3.59
F563-1	111	17.7	11.18
F563V1	30	7.4	1.57
F563V2	111	9.2	11.18
F564V4	40	~2	2.42
F565V2	51	8.4	3.48
F567-2	64	11.3	4.90
F568-1	119	14.9	12.42
F568-3	120	16.5	12.57
F568V1	124	19	13.21
F571-8	133	15.6	14.67
F571V1	73	14.6	5.97
F571V2	45	3.7	2.89
F574-1	100	15.4	9.56
F574-2	40	10.7	2.42
F577V1	30	8.9	1.57
F579V1	100	17.3	9.56
F583-1	85	14.6	7.49
F583-4	67	10	5.25
U0128	131	42.3	14.34
U1230	102	34.7	9.85
U5005	99	27.8	9.42
U5750	75	21.8	6.21
U5999	155	15.3	18.46

Table 1.The rotational speed, their radius and the corresponding crossover radius for LSB galaxies.(Taken from[26])

Name	Vmax	Rmax(Kpc)	Rcross(Mpc)
DDO154	48	7.6	3.18
DDO168	55	3.4	3.90
DDO170	66	9.6	5.13
N55	87	10.2	7.76
N257	108	9.9	10.73
N300	97	10.6	9.14
N801	222	58.7	31.64
N1003	115	31.3	11.79
N1530	79	8.3	6.72
N2403	136	19.5	15.17
N2841	323	81.1	55.52
N2903	201	24.2	27.25
N2998	214	46.6	29.94
N3109	67	8.2	5.25
N3198	157	29.9	18.81
N5033	222	35.4	31.64
N5533	273	74.4	43.14
N5585	92	9.6	8.44
N6503	121	22.2	12.73
N6674	266	64.5	41.49
N7331	241	36.7	35.78
U2259	90	7.6	8.17
U2885	298	72.5	49.20

Table 2: The rotational speed, their radius and the corresponding crossover radius for HSB galaxies (Taken from [26])

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